

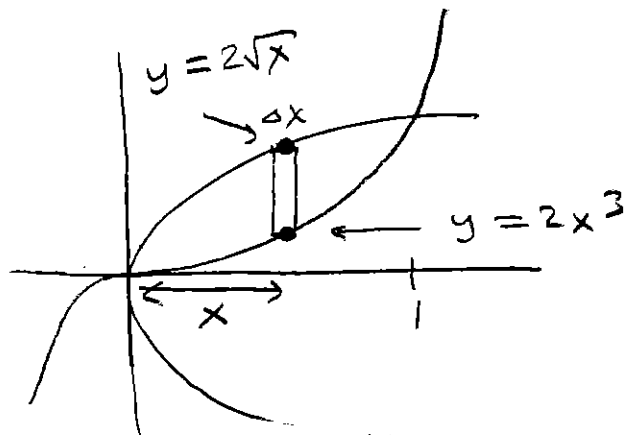
Closing Wed: HW\_3A,3B,3C (6.1-6.3)

Exam 1 is Thurs (4.9, 5.1-5.5, 6.1-6.3)

Entry Task:

Find the area of the region bounded by  $4x = y^2$  and  $y = 2x^3$  in 2 ways:

- (i) Using  $dx$
- (ii) Using  $dy$



$$\int_0^1 2\sqrt{x} - 2x^3 dx$$
$$\frac{4}{3} x^{3/2} - \frac{2}{4} x^4 \Big|_0^1$$

$$\frac{4}{3} - \frac{1}{2} = \frac{8}{6} - \frac{3}{6} = \boxed{\frac{5}{6}}$$

$$4x = y^2 \Leftrightarrow \begin{cases} y = -2\sqrt{x} \\ y = 2\sqrt{x} \end{cases}$$

$$x = \left(\frac{1}{2}y\right)^{1/3} \Leftrightarrow y = 2x^3$$

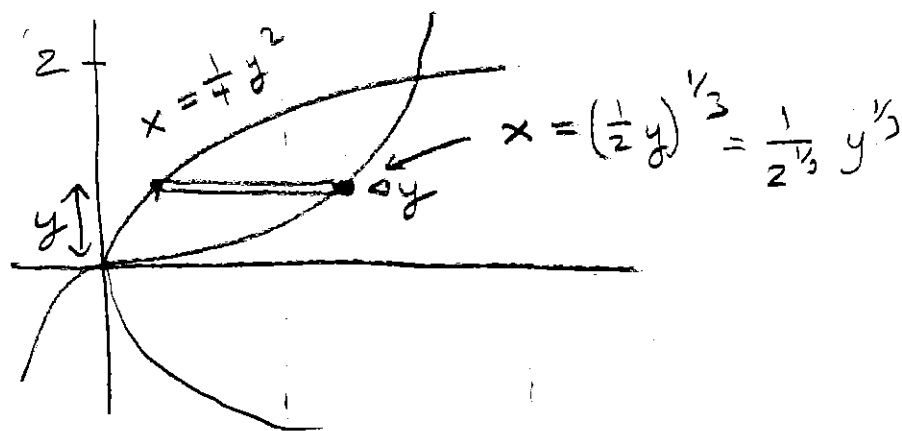
INTERSECTIONS  $\Rightarrow 4x = (2x^3)^2$

$$4x = 4x^6$$

$$x = x^6$$

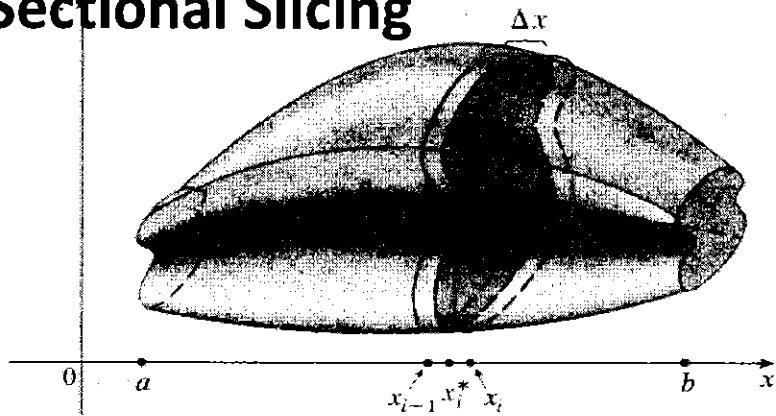
$$0 = x^6 - x = x(x^5 - 1)$$

$$x = 0 \text{ or } x = 1$$



$$\int_0^2 \left(\frac{1}{2}\right)^{1/3} y^{1/3} - \frac{1}{4} y^2 dy$$
$$= \left(\frac{1}{2}\right)^{1/3} \frac{3}{4} y^{4/3} - \frac{1}{12} y^3 \Big|_0^2$$
$$= \left(\frac{1}{2}\right)^{1/3} \frac{3}{4} (2)^{4/3} - \frac{1}{12} (2)^3$$
$$= \frac{6}{4} - \frac{8}{12} = \frac{3}{2} - \frac{2}{3} = \frac{9}{6} - \frac{4}{6} = \boxed{\frac{5}{6}}$$

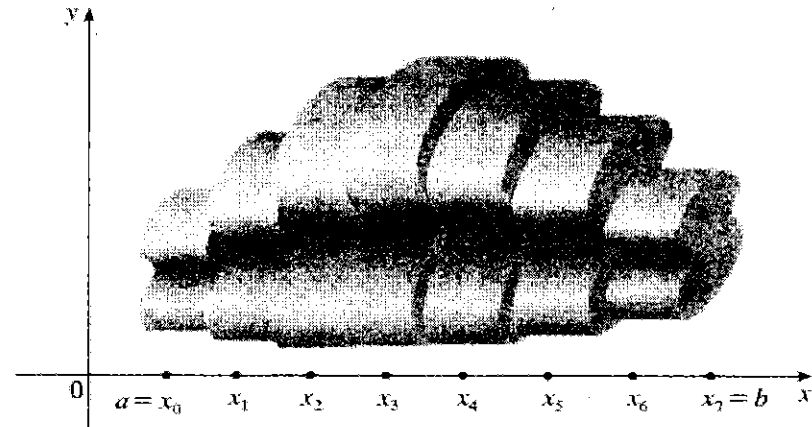
## 5.2 Finding Volumes Using Cross-Sectional Slicing



If we can find the general formula,  $A(x_i)$ , for the area of a cross-sectional slice, then we can approximate volume by:

Volume of one slice  $\approx A(x_i) \Delta x$

Total Volume  $\approx \sum_{i=1}^n A(x_i) \Delta x$



This approximation gets better and better with more subdivisions, so

$$\text{Exact Volume} = \lim_{n \rightarrow \infty} \sum_{i=1}^n A(x_i) \Delta x$$

We conclude


$$\text{Volume} = \int_a^b A(x) dx =$$


$$\int_a^b \text{"Cross-sectional area formula"} dx$$


## Volume using cross-sectional slicing

1. Draw region. Cut **perpendicular** to rotation axis. Label  $x$  if that cut crosses the  $x$ -axis (and  $y$  if  $y$ -axis). Label **everything** in terms this variable.

2. Formula for cross-sectional area?

*disc:* Area =  $\pi(\text{radius})^2$  

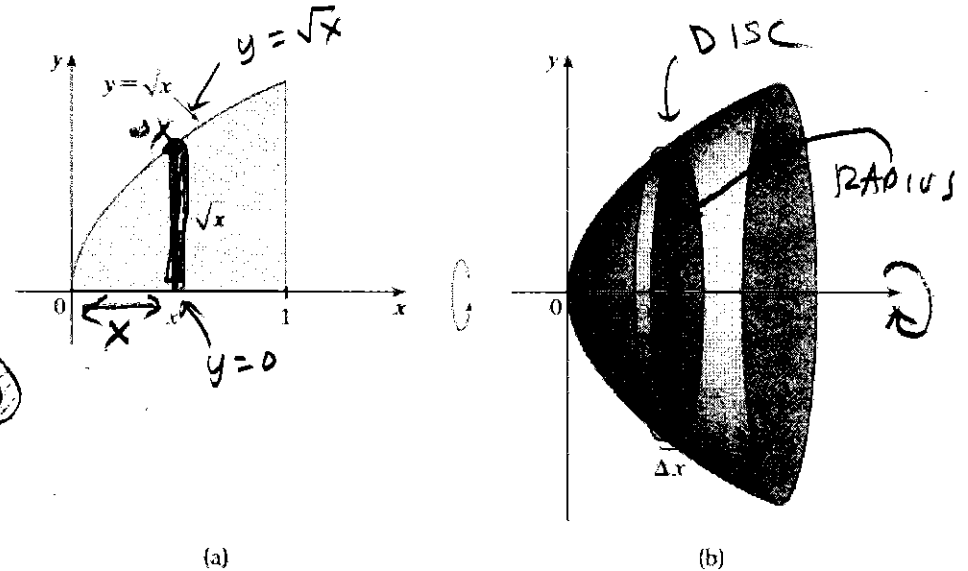
*washer:* Area =  $\pi(\text{outer})^2 - \pi(\text{inner})^2$  

*square:* Area = (Height)(Length) 

*triangle:* Area =  $\frac{1}{2}$  (Height)(Length)

3. Integrate the area formula.

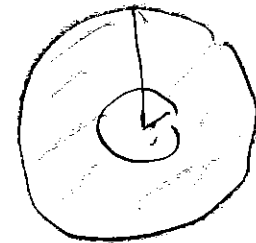
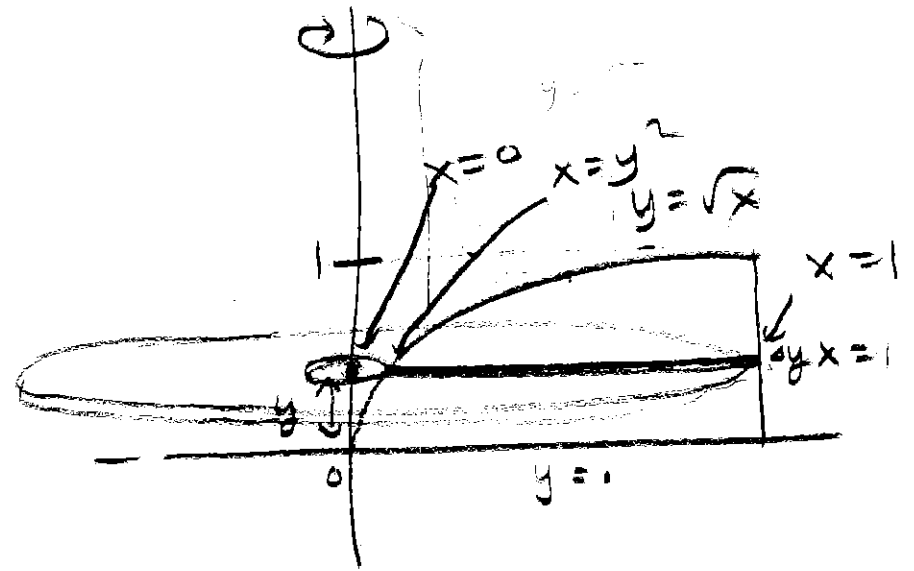
*Example:* Consider the region,  $R$ , bounded by  $y = \sqrt{x}$ ,  $y = 0$ , and  $x = 1$ . Find the volume of the solid obtained by rotating  $R$  about the  **$x$ -axis**.



$$\begin{aligned} & \int_0^1 \pi (\text{RADIUS})^2 dx \\ &= \int_0^1 \pi (\sqrt{x})^2 dx \\ &= \pi \int_0^1 x dx = \pi \left[ \frac{1}{2} x^2 \right]_0^1 \\ &= \boxed{\frac{\pi}{2}} \end{aligned}$$

**Example:** Consider the region,  $R$ , bounded by  $y = \sqrt{x}$ ,  $y = 0$ , and  $x = 1$ . Find the volume of the solid obtained by rotating  $R$  about the  **$y$ -axis**.

$$\begin{aligned}
 & \int_0^1 \pi (1)^2 - \pi (y^2)^2 dy \\
 &= \pi \int_0^1 1 - y^4 dy \\
 &= \pi \left( y - \frac{1}{5} y^5 \right) \Big|_0^1 \\
 &= \pi \left( 1 - \frac{1}{5} \right) = \boxed{\frac{4\pi}{5}}
 \end{aligned}$$



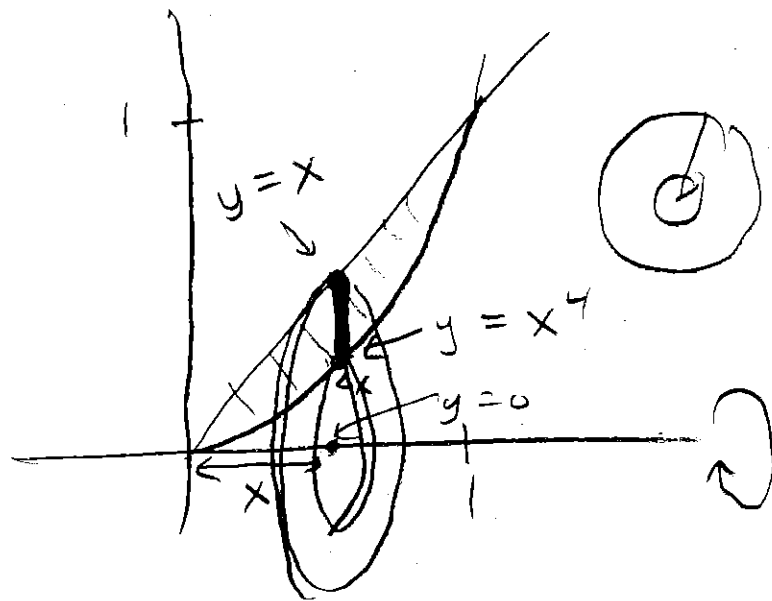
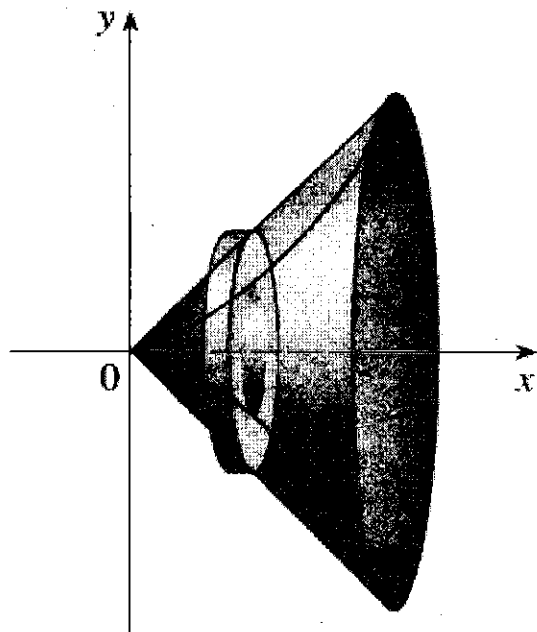
Example: Consider the region, R, bounded by  $y = x$  and  $y = x^4$ . Find the volume of the solid obtained by rotating R about the **x-axis**.

$$\int_0^1 \pi (x)^2 - \pi (x^4)^2 dx$$

$$\pi \int_0^1 x^2 - x^8 dx$$

$$\pi \left( \frac{1}{3} x^3 - \frac{1}{9} x^9 \right) \Big|_0^1$$

$$\pi \left( \frac{1}{3} - \frac{1}{9} \right) = \boxed{\frac{2\pi}{9}}$$



Example: Consider the region,  $R$ , bounded by  $y = x$  and  $y = x^4$ .  $R$  is the same as the last example).

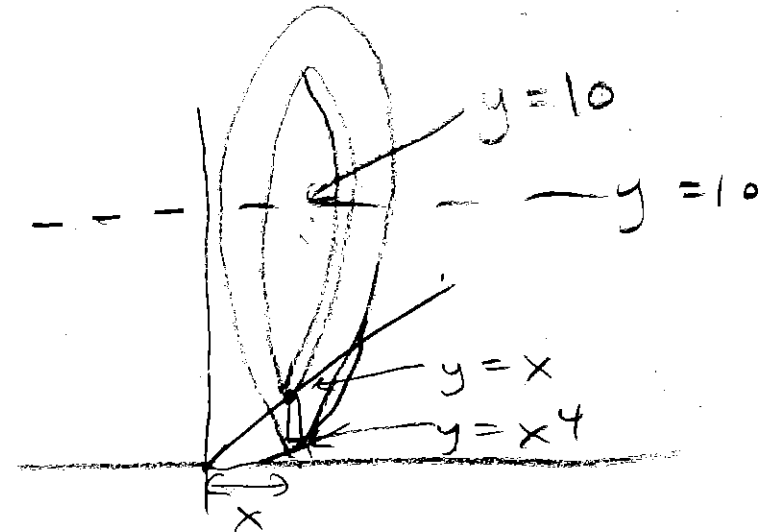
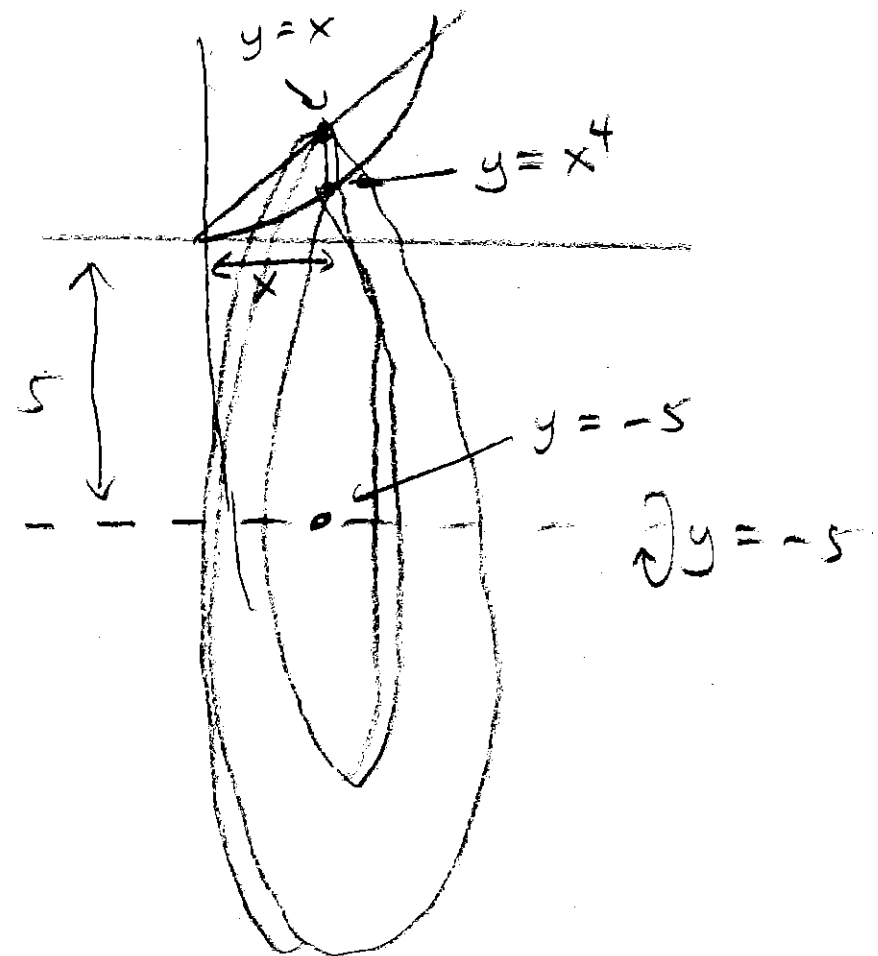
(a) Now rotate about the horizontal line  $y = -5$ . What changes?

$$\int_0^1 \pi (x - (-5))^2 - \pi (x^4 - (-5))^2 dx$$

$$\pi \int_0^1 (x + 5)^2 - (x^4 + 5)^2 dx$$

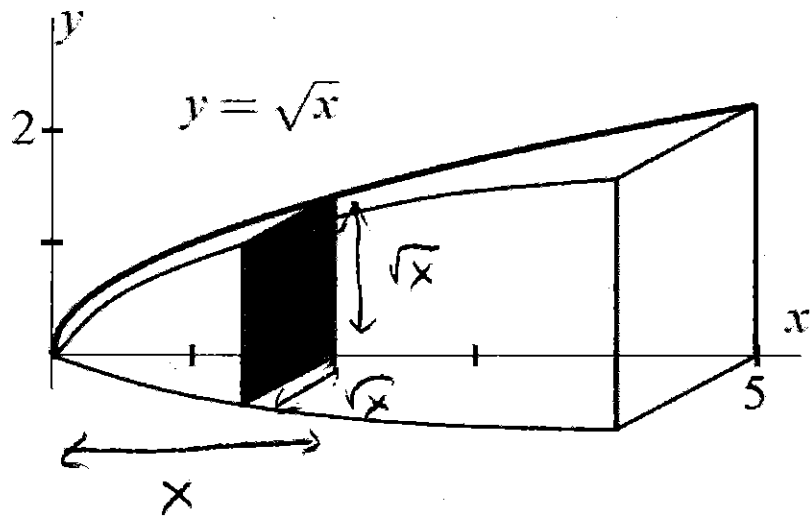
(b) Now rotate about the horizontal line  $y = 10$ . What changes?

$$\int_0^1 \pi (10 - x^4)^2 - \pi (10 - x)^2 dx$$



Example:

From an old final and homework)  
Find the volume of the solid shown.  
The cross-sections are squares.



$$\int_0^5 (\text{HEIGHT})(\text{LENGTH}) dx$$

$$\int_0^5 \sqrt{x} \cdot \sqrt{x} dx$$

$$\int_0^5 x dx = \frac{1}{2} x^2 \Big|_0^5 = \frac{1}{2} 25 = \boxed{\frac{25}{2}}$$

## Summary (Cross-sectional slicing):

1. Draw Label
2. Cross-sectional area?
3. Integrate area.

## This method has a major limitation:

5.2 method about  $x$ -axis, must use  $dx$ .

5.2 method about  $y$ -axis, must use  $dy$ .

What if the regions is rotated about the  $x$ -axis and we need to use  $dy$ ?  
or about  $y$ -axis and we need  $dx$ ?)

**In these cases, 6.2 “Cross-sectional slicing” wouldn’t work!**

We need another method.

That is what we will do in 6.3.



Close Wed: HW\_3A,3B,3C

(complete sooner!)

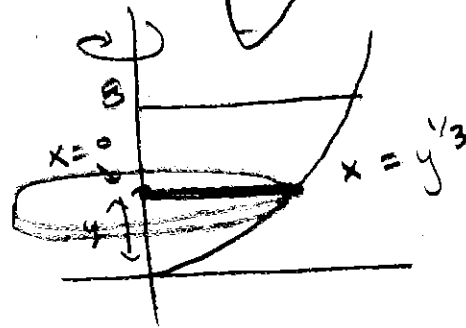
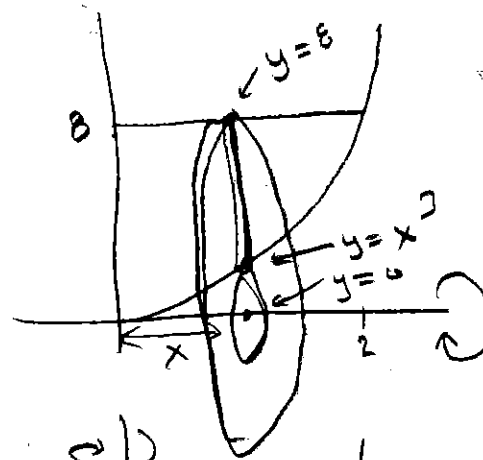
Exam 1 is Thurs (4.9, 5.1-5.5, 6.1-6.3)

Entry Task:

Consider the region  $R$  bounded by  $y = x^3$ ,  $y = 8$ , and  $x = 0$ .

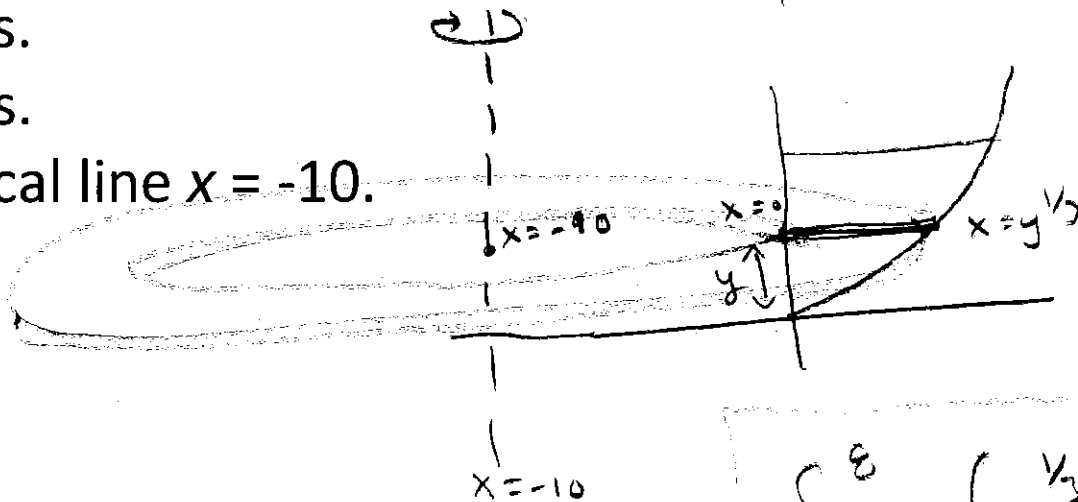
Set up the integrals that would give the volume of the solid obtained by rotating  $R$  about the ...

- (a) ...  $x$ -axis.
- (b) ...  $y$ -axis.
- (c) ... vertical line  $x = -10$ .



$$\int_0^2 \pi(8^2 - \pi(x^3)^2) dx$$
$$\pi \int_0^2 64 - x^6 dx$$

$$\int_0^8 \pi (y^{1/3})^2 dy$$
$$= \pi \int_0^8 y^{2/3} dy$$



$$\int_0^8 \pi (y^{1/3} + 10)^2 - \pi(10)^2 dy$$